

A Correction Note: Attractive Nearest Neighbor Spin Systems on the Integers

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Abstract

In this note, we discuss a proof in T. Liggett's work on attractive translation invariant nearest neighbor spin systems on the integers. A correction to a wrong estimate is provided.

1 Introduction

For the purposes of this note, an attractive translation invariant nearest neighbor spin system is a certain kind of Feller process defined on the compact state space $X = \{0, 1\}^{\mathbb{Z}}$ with rates satisfying the attractiveness inequalities, and depending only on the nearest neighbors. (See [2], p.144-145 for more details.)

T. Liggett proves in [1] and [2], (p. 152 Theorem 3.13) the following theorem:

Theorem 1. *All attractive translation invariant nearest neighbor spin systems on the integers have only the minimal and maximal invariant measures (ordered stochastically) as the extremal invariant measures.*

We discuss the estimate in [2] of lemma 3.7 part (e). It is wrong, but the similar estimate $\epsilon \int g_{m,n}^{l+1} d\nu \leq (4Kl + 2\epsilon) \int g_{m,n}^l d\nu$ is valid. This change does not affect the proof moving forward, because the only time the estimate is used is in the proof of lemma 3.10 in [2], where it is used to derive that $\sup_{m \leq n} \int g_{m,n}^l d\nu < \infty$. This fact still holds with the new estimate. The same line of reasoning appears in [1] within the proof of lemma 2.2 there.

We will refer to [2] from now on.

2 A Correction

The problem with the estimate as written is at the top of p.150 of [2] in the discussion of bounding below the positive contribution to $\tilde{\Omega} g_{m,n}^l$. The argument there fails to consider the possibility that change in the γ coordinate at any of the x_i among the left and right endpoints in the $l + 1$ length intervals may not only create a length l interval, but also destroy an adjacent length l interval. But this can only happen at at most $2g_{m,n}^l$ such sites, i.e. the left and right

neighbors of intervals of length l . The bound below on the rate of the type of flip in question is still correctly stated as ϵ so as long as we replace $\epsilon(g_{m,n}^{l+1})$ by $\epsilon(g_{m,n}^{l+1} - 2g_{m,n}^l)$ the estimate is correct. This results in the display reading

$$\tilde{\Omega}g_{m,n}^l \geq \epsilon(g_{m,n}^{l+1} - 2g_{m,n}^l) - 4Klg_{m,n}^l.$$

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References

- [1] Liggett, T. (1978). Attractive Nearest Neighbor Spin Systems on the Integers. Ann. Probability Vol. 6 No. 4, p.629-636.
- [2] Liggett, T. (1985). Interacting Particle Systems. Springer Berlin Heidelberg 2005. p.143-152 and Ch. 1.